Chain rule quiz answers

Differentiate the following :

① $y = (x+2)^3$

② $y = (3x+2)^4$

③ $y = \dfrac{1}{5x+1}$

④ $y = \dfrac{3}{5x^2+4}$

⑤ $y = \dfrac{2}{(3x+1)^2}$

④ $y = \dfrac{3}{u}$ where $u = 5x^2+4$

$y = 3u^{-1}$ $\dfrac{du}{dx} = 10x$

$\dfrac{dy}{du} = -3u^{-2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$= -3u^{-2} \times 10x$

$= -30x \, u^{-2}$

$= \dfrac{-30x}{u^2}$

$= \dfrac{-30}{(5x^2+4)^2}$

⑤

$y = 2(3x+1)^{-2}$

$y = 2u^{-2}$ where $u = 3x+1$

$\dfrac{dy}{du} = -4u^{-3}$ $\dfrac{du}{dx} = 3$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Answers :

Using formula

① $y = u^3$ where $u = x+2$

$\dfrac{dy}{du} = 3u^2$ $\dfrac{du}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$= 3u^2 \times 1$

$= 3(x+2)^2$

② $y = u^4$ where $u = 3x+2$

$\dfrac{dy}{du} = 4u^3$ $\dfrac{du}{dx} = 3$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$= 4u^3 \times 3$

$= 4(3x+2)^3 \times 3$

$= 12(3x+2)^3$

③ $y = \dfrac{1}{u}$ where $u = 5x+1$

$y = u^{-1}$ $\dfrac{du}{dx} = 5$

$\dfrac{dy}{du} = -1u^{-2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$= -1u^{-2} \times 5$

$= -5u^{-2}$

$= \dfrac{-5}{u^2}$

$= \dfrac{-5}{(5x+1)^2}$

By inspection

① $y = (x+2)^3$

$y' = 3(x+2)^2 \times 1$

② $y = (3x+2)^4$

$y' = 4(3x+2)^3 \times 3$

$= 12(3x+2)^3$

③ $y = (5x+1)^{-1}$

$y' = -1(5x+1)^{-2} \times 5$

$= -5(5x+1)^{-2}$

$= \dfrac{-5}{(5x+1)^2}$

④ $y = 3(5x^2+4)^{-1}$

$y' = -3(5x^2+4)^{-2} \times 10x$

$= \dfrac{-30x}{(5x^2+4)^2}$

⑤ $y = 2(3x+1)^{-2}$

$y' = -4(3x+1)^{-3} \times 3$

$= \dfrac{-12}{(3x+1)^3}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -4u^{-3} \times 3$$

$$= \frac{-12}{u^3}$$

$$= \frac{-12}{(3x+1)^3}$$