Integral Calculus lesson 2 - More complex integrals

Magic Monk Tutorials

Important Rule 1: $\int ax^n \ dx = \dfrac{ax^{n+1}}{n+1} + c.$

For Questions 1.1 to 1.4, transform the integrals to the form of $\int ax^n \ dx$ using index laws.

For Questions 1.5 to 1.6, expand to simplify the integral into something easier.

1 Evaluate the following integrals:

1.1 $\displaystyle\int \frac{2}{x^3} \ dx$

$$\int \frac{2}{x^3} \ dx = \int 2x^{-3} \ dx$$

Now apply Rule 1 as above.

$$\int 2x^{-3} \ dx = \frac{2x^{-3+1}}{-3+1} + c = -x^{-2} + c$$

This can be further simplified to $\displaystyle\int \frac{2}{x^3} \ dx = -\frac{1}{x^2} + c$

1.2 $\displaystyle\int \sqrt{x} \ dx$

$$\int \sqrt{x} \ dx = \int x^{1/2} \ dx$$

Applying rule 1,

$$\int x^{1/2} \ dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}x^{\frac{3}{2}} + c.$$

1.3 $\displaystyle\int \frac{-5}{2x^{7/2}} \ dx$

Factor out the constants and simplify using index laws.

$$\int \frac{-5}{2x^{7/2}} \ dx = \frac{-5}{2} \int x^{-\frac{7}{2}} \ dx \quad \text{Apply rule 1.}$$

$$\frac{-5}{2} \int x^{-\frac{7}{2}} \ dx = \frac{-5}{2} \cdot \frac{x^{\frac{-7}{2}+1}}{\frac{-7}{2}+1} + c = \frac{-5}{2} \cdot \frac{x^{\frac{-5}{2}}}{\frac{-5}{2}} + c$$

Simplify Further.

$$\int \frac{-5}{2x^{7/2}} \ dx = \frac{1}{x^{\frac{5}{2}}} + c$$

1.4 $\displaystyle\int x^{-4}\ dx$

Apply rule 4 directly.

$$\int x^{-4}\ dx = \frac{x^{-4+1}}{-4+1} + c = -\frac{x^{-3}}{3} + c$$

1.5 $\displaystyle\int x\,(x+1)^2\ dx$

Expand to simplify the integral.

$$\int x\,(x+1)^2\ dx = \int x\,(x+1)\,(x+1)\ dx = \int x\,(x^2 + 2x + 1)\ dx = \int x^3 + 2x^2 + x\ dx$$

Integrate now with rule 1 (see worksheet 1 for more examples).

$$\int x^3 + 2x^2 + x\ dx = \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + c$$

1.6 $\displaystyle\int \frac{\left(x^2 - x\right)^2}{x^2}\ dx$

Use index laws to simplify the integral.

$$\int \frac{\left(x^2 - x\right)^2}{x^2}\ dx = \int \left(\frac{x^2 - x}{x}\right)^2\ dx = \int (x-1)^2\ dx = \int (x-1)\,(x-1)\ dx$$
$$= \int x^2 - 2x + 1\ dx$$

Integrate using rule 1.

$$\int x^2 - 2x + 1\ dx = \frac{x^3}{3} - x^2 + x + c$$