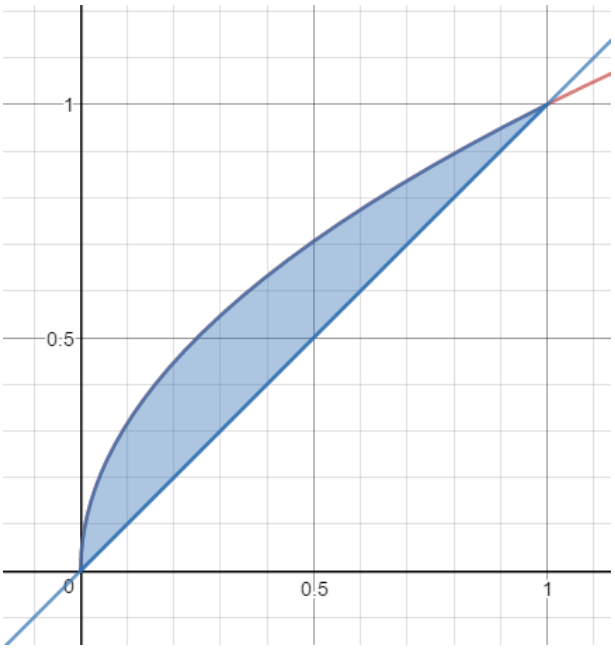
Integral Calculus lesson 7 - Area bounded by 2 curves

Magic Monk Tutorials

1 Evaluate the following using the method in Integration Lesson 7:

1.1 The shaded area between the curves $y = \sqrt{x}$ (in red) and $y = x$ (in blue)



Solve intercepts. $\sqrt{x} = x$ at $x = 0$ and 1. Write the Area in integral form, where $y = \sqrt{x}$ is the top function in the bounds.

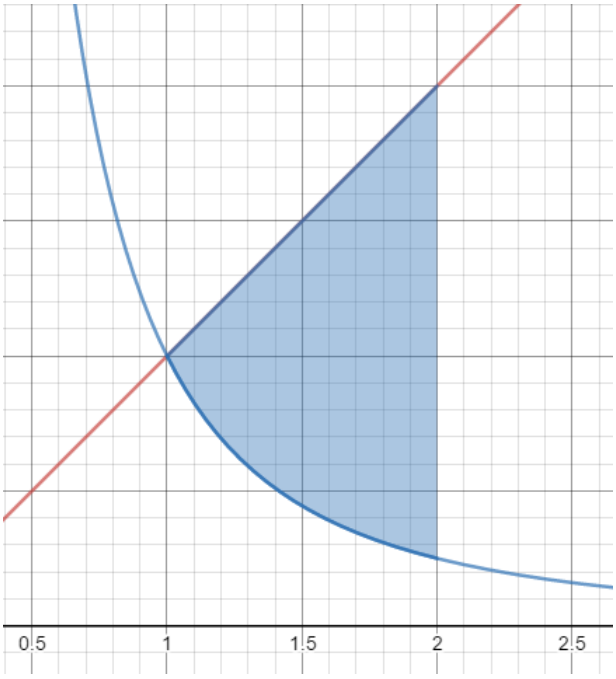
$$\text{Area} = \int_0^1 \sqrt{x}\, dx - \int_0^1 x\, dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ units squared.}$$

1.2 The shaded area between the curves $y = x^2$ (in red) and $y = x^3$ (in blue)

Solve intercepts. $x^2 = x^3$ at $x = 0$ and 1. Write the Area in integral form, where $y = x^2$ is the top function in the bounds.

$$\text{Area} = \int_0^1 x^2\,dx - \int_0^1 x^3\,dx = \left[\frac{x^3}{3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \text{ units squared.}$$

1.3 The shaded area between the curves $y = x$ (in red) and $y = \dfrac{1}{x^2}$ (in blue) between x=1 and 2

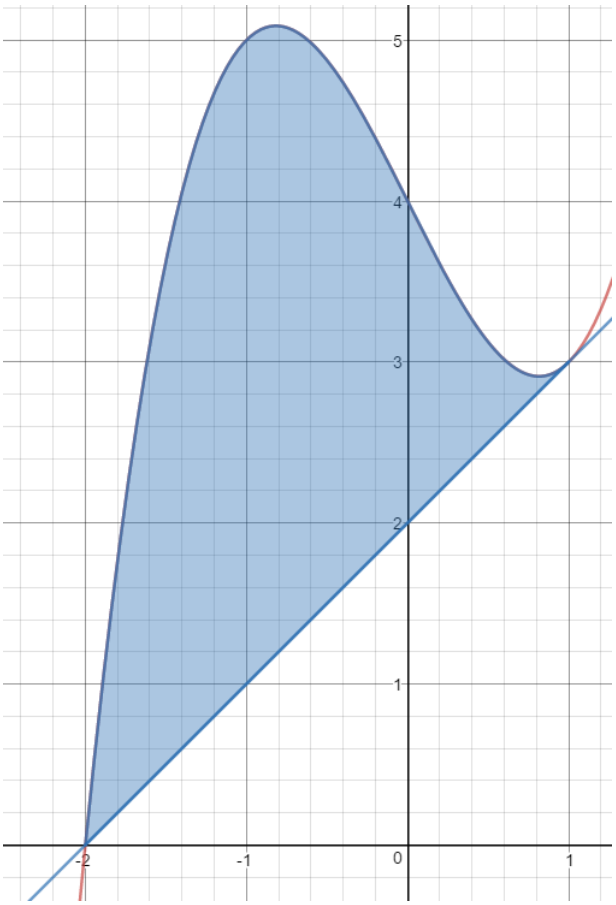


We are told the bounds are $x = 1$ and 2. Write the Area in integral form, where $y = x$ is the top function in the bounds.

$$\text{Area} = \int_1^2 x\,dx - \int_1^2 x^{-2}\,dx = \left[\frac{x^2}{2}\right]_1^2 - \left[\frac{x^{-1}}{-1}\right]_1^2 = \left[\frac{4}{2} - \frac{1}{2}\right] - \left[\frac{2^{-1}}{-1} - \frac{1^{-1}}{-1}\right] = \frac{3}{2} - \frac{1}{2} = 1$$

units squared.

1.4 The shaded area between the curves $y = x^3 - 2x + 4$ (in red) and $y = x + 2$ (in blue)



From looking at the graph, the intersects are $x = -2$ and 1. Write the Area in integral form, where $y = x^3 - 2x + 4$ is the top function in the bounds.

$$\text{Area} = \int_{-2}^{1} x^3 - 2x + 4 \, dx - \int_{-2}^{1} x + 2 \, dx = \left[\frac{x^4}{4} - x^2 + 4x\right]_{-2}^{1} - \left[\frac{x^2}{2} + 2x\right]_{-2}^{1}$$

$$= \left[\frac{1^4}{4} - 1^2 + 4 - \frac{(-2)^4}{4} + (-2)^2 - 4(-2)\right] - \left[\frac{1^2}{2} + 2 - \frac{(-2)^2}{2} - 2(-2)\right] = \left[\frac{1}{4} - 1 + 4 - \frac{16}{4} + 4 + 8\right] -$$

$$\left[\frac{1}{2} + 2 - \frac{4}{2} + 4\right] = \frac{-15}{4} + 15 - \frac{1}{2} - 4 = \frac{27}{4} \text{ units squared.}$$