Linear transformations with Matrices lesson 10 - Reflection in the line y=x

Magic Monk Tutorials

1 Reflect the point $P = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ about the line $y = x$.

Use the general formula for linear transformations with the transformation matrix

$R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

$\begin{pmatrix} x' \\ y' \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

2 Find the transformation matrix corresponding with reflection in the line $y = -x$

We wish to find where the transformation maps the points $(1, 0)$ and $(0, 1)$.
By drawing a similar graph to one shown in the tutorial video, one can see that $(1, 0) \mapsto (-1, 0)$ and $(0, 1) \mapsto (-1, 0)$. Therefore our transformation matrix is
$R = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

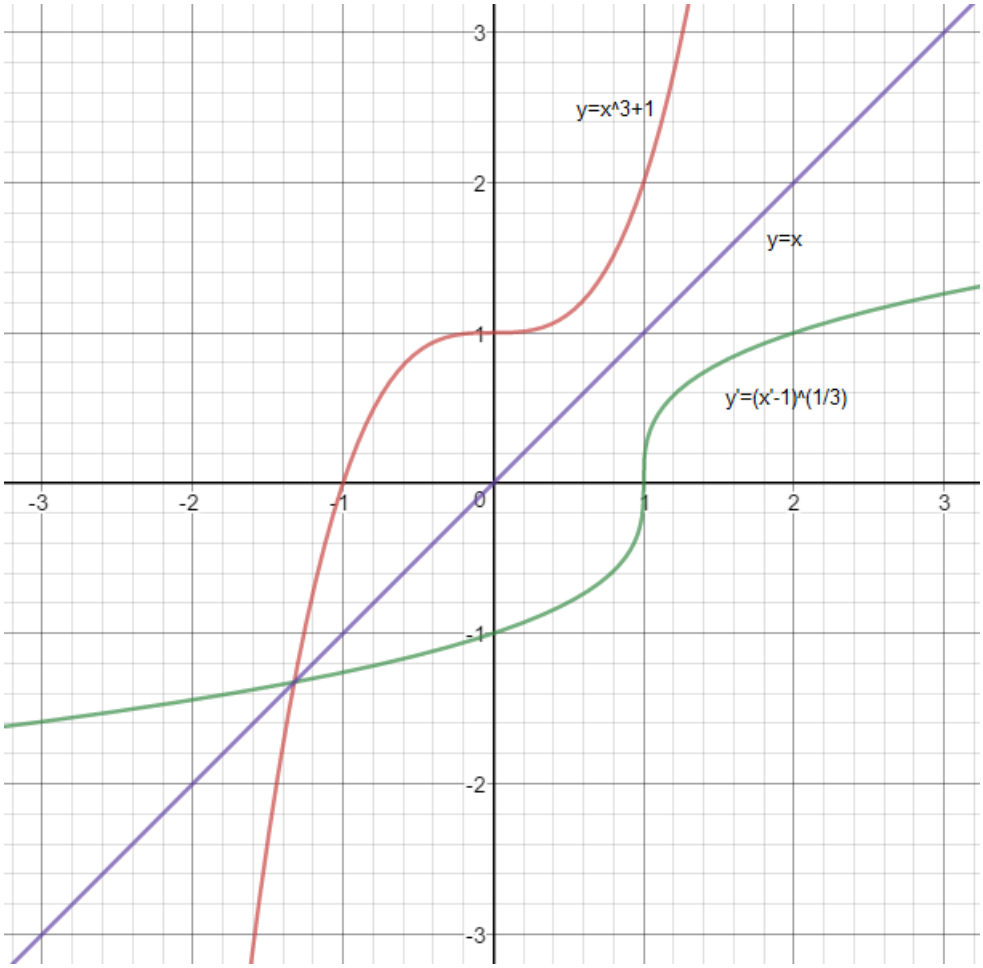
3 Reflect the line $y = x^3 + 1$ in the line $y = x$. Plot this resulting line below in the $x - y$ plane.

Use the general formula for linear transformations with the transformation matrix

$\begin{pmatrix} x' \\ y' \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$

Therefore $y = x'$ and $x = y'$. Substitute these into our equation $y = x^3 + 1$ and rearrange for y.
$y = x^3 + 1$
$x' = y'^3 + 1$
$x' - 1 = y'^3$
$y' = (x' - 1)^{1/3}$.

This is plotted below. Note that we have found the inverse function of $y = x^3 + 1$ by applying the transformation.


y=x^3+1
y=x
y'=(x'-1)^(1/3)
3
2
1
0
-1
-2
-3
-3
-2
-1
1
2
3