Linear transformations with Matrices lesson 2 - Translation of a curve

Magic Monk Tutorials

1 Translate the curve $y = 2x - 3$ by the point $p_1 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and plot it in the x-y plane.

As discussed in the video, translation of a curve by a point is done using the following formula.

$$\begin{pmatrix} x \\ y \end{pmatrix} + p_1 = \begin{pmatrix} x' \\ y' \end{pmatrix}.$$

We can substitute in our p_1 and simplify this expression.

$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' + 2 \\ y' - 3 \end{pmatrix}$$

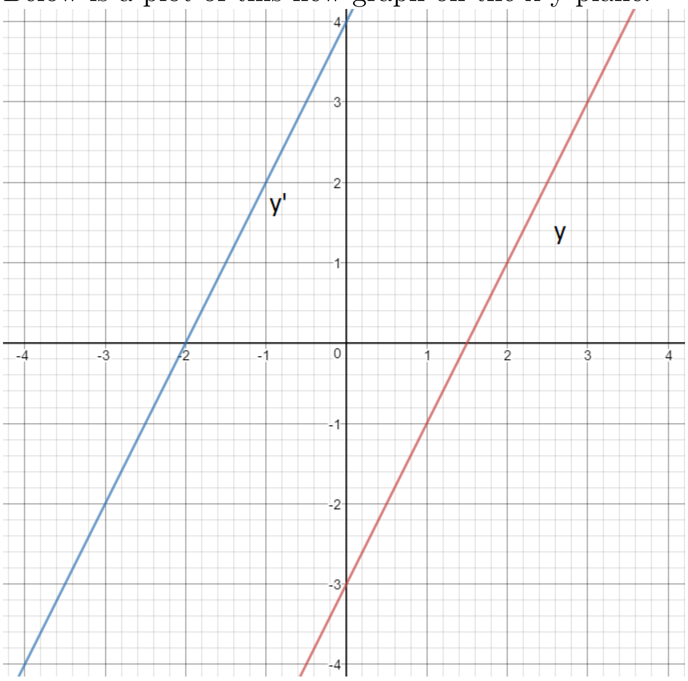
Now we can substitute x for $x' + 2$ and y for $y' - 3$ and rearrange to get our resulting function.

$y = 2x - 3$

$(y' - 3) = 2\,(x' + 2) - 3$

$y' = 2x' + 4$

Below is a plot of this new graph on the x-y plane.



2 Translate the curve $y = x^2 + 2x + 2$ by the point $T = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, simplify the resulting function and plot the result in the x-y plane.

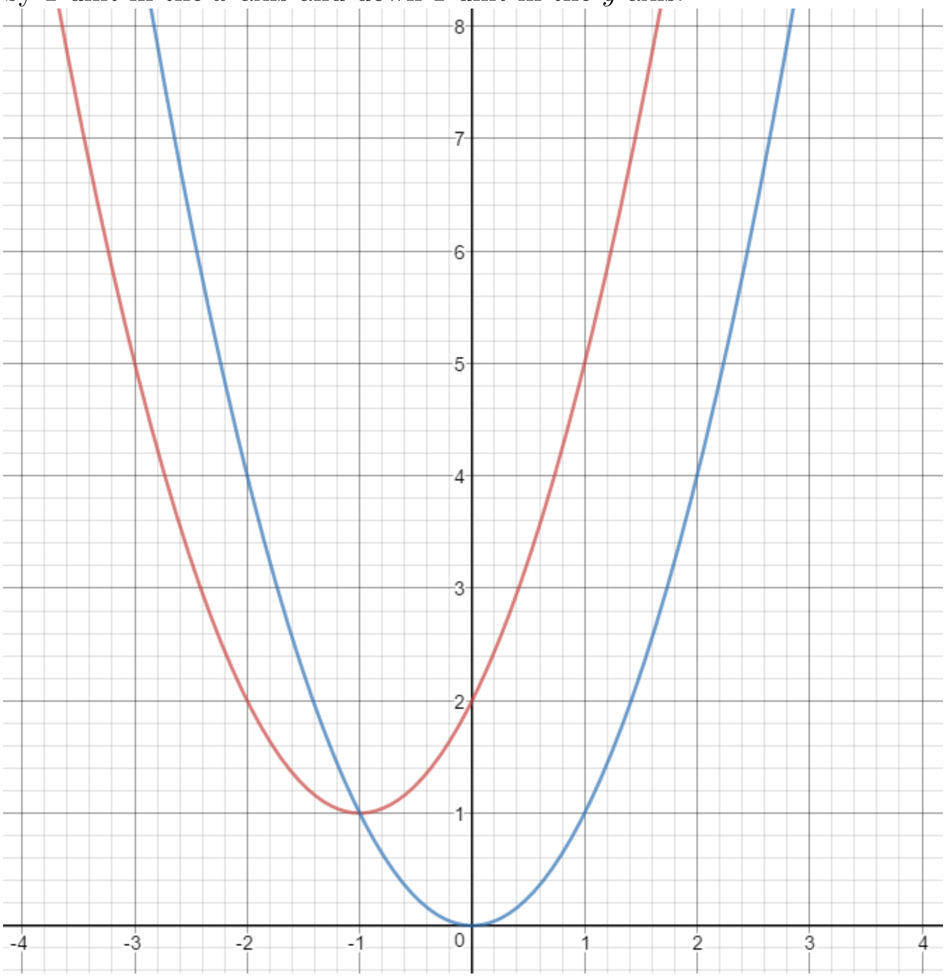
Use the following formula for translating a curve.

$$\begin{pmatrix} x \\ y \end{pmatrix} + T = \begin{pmatrix} x' \\ y' \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} x' - 1 \\ y' + 1 \end{pmatrix}$$

As before, substitute x for $x' - 1$ and y for $y' + 1$ in the original curve.

$$y = x^2 + 2x + 2$$
$$y' + 1 = (x' - 1)^2 + 2(x' - 1) + 2$$
$$y' = x'^2 - 2x' + 1 + 2x' - 2 + 2 - 1$$
$$y' = x'^2$$

The plot can be seen below. Note that all we have done is shifted the original curve right by 1 unit in the x axis and down 1 unit in the y axis.



3 Find a point that translates the curve $y = \frac{1}{x}$ so that it passes through the point $x = y = 4$.

We wish to find a, b so that a point (x, y) exists such that $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

Rearranging gives

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4-a \\ 4-b \end{pmatrix}$$

Now substituting this into our curve $y = \frac{1}{x}$ results in

$$4 - a = \frac{1}{4-b}$$
$$(4-a)(4-b) = 1$$

Now we may read of a solution, $a = b = 3$ satisfies the above equation. So our transformation is $T = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

Note that there are infinitely many solutions to the above equation, $a = b = 5$ being another solution we could have chosen.

4 Translate the curve $y = \sin(x)$ by the point $T = \begin{pmatrix} -\pi/2 \\ 0 \end{pmatrix}$, and plot the result in the x-y plane.

As before, $\begin{pmatrix} x \\ y \end{pmatrix} + T = \begin{pmatrix} x' \\ y' \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' + \pi/2 \\ y' \end{pmatrix}$$

And we have $y' = \sin(x' + \pi/2)$ which is equivalent to $y' = \cos(x')$.