Linear transformations with Matrices lesson 4 - Finding the transformation matrix

Magic Monk Tutorials

1 Find the transformation matrix associated with reflection on the x axis.

Find 2 example points to substitute into the transformation formula.

$(1, 1) \mapsto (1, -1)$
$(-1, 1) \mapsto (-1, -1)$

Remembering the formula for the linear transformation,

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

Now if we substitute our points, we get the two following systems of equations. From point 1,

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a - b \\ c - d \end{pmatrix}$$

so $a = 1 + b$, $c = 1 + d$. Now from point 2,

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -a - b \\ -c - d \end{pmatrix}$$

so $a = 1 - b$, $c = -d - 1$. We can now solve these 2 simultaneous equations. The result is $b = 0, a = 1, d = -1, c = 0$. This means our transformation matrix is

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2 Find the transformation matrix associated with the following mapping of points.

$(1, 1) \mapsto (2, 1)$
$(-1, 2) \mapsto (1, 2)$

As before, our substitute our two points into the transformation formula, $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

For point 1,

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} a + b \\ c + d \end{pmatrix}$$

So we have $a = 2 - b$, $c = 1 - d$. For point 2,

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -a + 2b \\ -c + 2d \end{pmatrix}$$

So we also have $a = 2b - 1$, $c = 2d - 2$. Now we may solve the two simultaneous equations, which results in $b = 1, a = 1, d = 1, c = 0$. So we have the following transformation matrix, this matrix is known as the shear matrix.

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

3 Sketch the function $y = 2x$ after it has been transformed by the transformation matrix $T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ on the x-y plane.

Try some points.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$$

Plot these points and connect them in a straight line. Note this new line is $y = 3x$.