Linear transformations with Matrices lesson 6 - Rotate a point N degrees about the origin

Magic Monk Tutorials

1 State the transformation matrix for an anticlockwise rotation about the origin of:

1.1 45 degrees.

The transformation matrix for anticlockwise rotation is given by $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

Substitute $\theta = 45°$ and simplify. $R_{45°} = \begin{pmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{pmatrix}$

$$R_{45°} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

1.2 120 degrees.

Substitute $\theta = 120°$ into the R_θ formula above and simplify.

$$R_{120°} = \begin{pmatrix} \cos 120° & -\sin 120° \\ \sin 120° & \cos 120° \end{pmatrix}$$
$$R_{120°} = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

2 Find the transformation matrix for clockwise rotation about the origin.

See how to do the anticlockwise rotation matrix in the video for lesson 6. Note that a clockwise rotation of θ degrees is equivalent to an anticlockwise rotation of $-\theta$ degrees. Sub θ for $-\theta$ in the Rotation matrix formula above.

$$R_{-\theta} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$

We know the identities $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$. Therefore we can simplify the above. Our clockwise rotation matrix is now

$$R_{-\theta} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

3 Rotate the point $P = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ by 45 degrees anticlockwise.

We can use our result from question 1.1. Apply the transformation formula.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R_{45^\circ} P$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2/\sqrt{2} \\ 4/\sqrt{2} \end{pmatrix} \approx \begin{pmatrix} -1.41 \\ 2.83 \end{pmatrix}$$

4 Rotate the point $P = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ by 30 degrees clockwise.

Rotation by 30 degrees clockwise is the same as rotation by -30 degrees clockwise.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R_{-30^\circ} P$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(-30^\circ) & -\sin(-30^\circ) \\ \sin(-30^\circ) & \cos(-30^\circ) \end{pmatrix} \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -\sqrt{3} - 3/2 \\ 1 - 3\sqrt{3}/2 \end{pmatrix} \approx \begin{pmatrix} -3.23 \\ -1.60 \end{pmatrix}$$