

Matrices lesson 10 - Solving sim. equations with matrices

Magic Monk Tutorials

1 Solve the following matrix equations with matrices.

$$A = \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}, B = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix}$$

1.1 $AX + BX = C$

First, rearrange for the matrix X.

$$\begin{aligned} AX + BX &= C \\ (A + B)X &= C \\ X &= (A + B)^{-1}C \end{aligned}$$

Calculate $A + B$ in order to calculate $(A + B)^{-1}$.

$$\begin{aligned} A + B &= \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 12 \\ 3 & 8 \end{pmatrix} \\ (A + B)^{-1} &= \begin{pmatrix} 5 & 12 \\ 3 & 8 \end{pmatrix}^{-1} = \frac{1}{40 - 36} \begin{pmatrix} 8 & -12 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -3/4 & 5/4 \end{pmatrix} \end{aligned}$$

Substitute this back into the matrix equation.

$$X = \begin{pmatrix} 2 & -3 \\ -3/4 & 5/4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 2 - 15 & 2 - 18 \\ -3/4 + 25/4 & -3/4 + 15/2 \end{pmatrix} = \begin{pmatrix} -13 & -16 \\ 11/2 & 27/4 \end{pmatrix}.$$

1.2 $XA + XB = C$

First, rearrange for the matrix X.

$$\begin{aligned} XA + XB &= C \\ X(A + B) &= C \\ X &= C(A + B)^{-1} \end{aligned}$$

We know $(A + B)^{-1}$ from the previous question.].

$$(A + B)^{-1} = \begin{pmatrix} 2 & -3 \\ -3/4 & 5/4 \end{pmatrix}$$

Substitute this back into the matrix equation.

$$X = \begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ -3/4 & 5/4 \end{pmatrix} = \begin{pmatrix} 2 - 3/4 & -3 + 5/4 \\ 10 - 18/4 & -15 + 30/4 \end{pmatrix} = \begin{pmatrix} 5/4 & -7/4 \\ 11/2 & -15/2 \end{pmatrix}.$$

1.3 $A + XB = XC$

First, rearrange for the matrix X.

$$\begin{aligned}A + XB &= XC \\A &= XC - XB \\A &= X(C - B) \\A(C - B)^{-1} &= X\end{aligned}$$

Calculate $C - B$ in order to calculate $(C - B)^{-1}$.

$$\begin{aligned}C - B &= \begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 4 & 3 \end{pmatrix} \\(C - B)^{-1} &= \begin{pmatrix} -1 & -4 \\ 4 & 3 \end{pmatrix}^{-1} = \frac{1}{-3 + 16} \begin{pmatrix} 3 & 4 \\ -4 & -1 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} 3 & 4 \\ -4 & -1 \end{pmatrix}\end{aligned}$$

Substitute this back into the matrix equation.

$$X = \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix} \cdot \frac{1}{13} \begin{pmatrix} 3 & 4 \\ -4 & -1 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -4 & -1 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} 9 - 28 & 12 - 7 \\ 6 - 20 & 8 - 5 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} -19 & 5 \\ -14 & 3 \end{pmatrix}.$$

1.4 $XA + 2B = XC$

First, rearrange for the matrix X.

$$\begin{aligned}XA + 2B &= XC \\2B &= XC - XA \\2B &= X(C - A) \\2B(C - A)^{-1} &= X\end{aligned}$$

Calculate $C - A$ in order to calculate $(C - A)^{-1}$.

$$\begin{aligned}C - A &= \begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix} - \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} -2 & -6 \\ 3 & 1 \end{pmatrix} \\(C - A)^{-1} &= \begin{pmatrix} -2 & -6 \\ 3 & 1 \end{pmatrix}^{-1} = \frac{1}{-2 + 18} \begin{pmatrix} 1 & 6 \\ -3 & -2 \end{pmatrix} = \frac{1}{16} \begin{pmatrix} 1 & 6 \\ -3 & -2 \end{pmatrix}\end{aligned}$$

Substitute this back into the matrix equation.

$$X = 2 \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \frac{1}{16} \begin{pmatrix} 1 & 6 \\ -3 & -2 \end{pmatrix} = \frac{1}{8} \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 6 \\ -3 & -2 \end{pmatrix} = \frac{1}{8} \begin{pmatrix} 2 - 15 & 12 - 10 \\ 1 - 9 & 6 - 6 \end{pmatrix} = \frac{1}{8} \begin{pmatrix} -13 & 2 \\ -8 & 0 \end{pmatrix}.$$