Matrices lesson 11 - Gaussian Elimination part 1

Magic Monk Tutorials

1 Solve the following simultaneous equations with Gaussian Elimination.

1.1

$x + y = 3$
$2x + y = 2$
Put the above in matrix form.

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

We may now augment the above matrices to help simplify our working.

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 1 & 2 \end{array}\right)$$

$R_2 \to R_2 - 2R_1$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -1 & -4 \end{array}\right)$$

From row 2, we can see that $-1y = -4$ and therefore $y = 4$. From row 1 we have $x + y = 3$, therefore with $y = 4$ we have $x = -1$.

1.2

$5x + 2y = 4$
$2x + y = 1$
Put the above in matrix form.

$$\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

We may now augment the above matrices to help simplify our working.

$$\left(\begin{array}{cc|c} 5 & 2 & 4 \\ 2 & 1 & 1 \end{array}\right)$$

$R_1 \to 2R_1$
$R_2 \to 5R_1$

$$\left(\begin{array}{cc|c} 10 & 4 & 8 \\ 10 & 5 & 5 \end{array}\right)$$

$R_2 \to R_2 - R_1$

$$\left(\begin{array}{cc|c} 10 & 4 & 8 \\ 0 & 1 & -3 \end{array}\right)$$

From row 2, we can see that $y = -3$. From row 1 we have $10x + 4y = 8$, therefore with $y = -3$ we have $10x = 8 + 12$ and therefore $x = 2$.

1.3

$6x + 5y = 7$
$5x + 4y = 4$
Put the above in matrix form.

$$\begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

We may now augment the above matrices to help simplify our working.

$$\left(\begin{array}{cc|c} 6 & 5 & 7 \\ 5 & 4 & 4 \end{array}\right)$$

$R_1 \to 5R_1$
$R_2 \to 6R_1$

$$\left(\begin{array}{cc|c} 30 & 25 & 35 \\ 30 & 24 & 24 \end{array}\right)$$

$R_2 \to R_2 - R_1$

$$\left(\begin{array}{cc|c} 30 & 25 & 35 \\ 0 & -1 & -11 \end{array}\right)$$

From row 2, we can see that $y = 11$. From row 1 we have $30x + 25y = 35$, which is equivalent to $6x + 5y = 7$. Substituting our y into this we have $6x = 7 - 55$, which simplifies to $x = -8$.

1.4

$x + 2y = 1$
$8x + 7y = 3$ Put the above in matrix form.

$$\begin{pmatrix} 1 & 2 \\ 8 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

We may now augment the above matrices to help simplify our working.

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 8 & 7 & 3 \end{array}\right)$$

$R_2 \to R_2 - 8R_1$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -9 & -5 \end{array}\right)$$

From row 2, we can see that $y = 5/9$. From row 1 we have $x + 2y = 1$. Substituting our y into this we have $x = 1 - 10/9$, which simplifies to $x = -1/9$.