Matrices lesson 4 - multiplying a matrix by a matrix

Magic Monk Tutorials

1 Given the following matricies, decide if each of the multiplications are possible. If so, calculate them.:

$A = \begin{pmatrix} 1 & 3 \\ 2 & 3 \\ 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 2 \\ 5 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 5 & 2 \\ 4 & -3 & 3 \\ 2 & 1 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 3 & -2 & 7 \\ 8 & 1 & 2 \end{pmatrix}$, $E = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$,
$F = \begin{pmatrix} 2 & -3 \end{pmatrix}$, $G = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$, $H = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 4 & -4 \\ -3 & 5 & 2 \end{pmatrix}$

1.1 $A \cdot B$

Possible, A is 3x2 and B is 2x2. Resulting matrix will be a 3x2 matrix.
$A \cdot B = \begin{pmatrix} 1 & 3 \\ 2 & 3 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 2 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 4 + 3 \cdot 5 & 1 \cdot 2 + 3 \cdot 1 \\ 2 \cdot 4 + 3 \cdot 5 & 2 \cdot 2 + 3 \cdot 1 \\ 3 \cdot 4 + 5 \cdot -1 & 3 \cdot 2 + 1 \cdot -1 \end{pmatrix} = \begin{pmatrix} 19 & 5 \\ 23 & 7 \\ 7 & 5 \end{pmatrix}$

1.2 $B \cdot A$

Not Possible, B is 2x2 and A is 3x2.

1.3 $B \cdot D$

Possible, B is 2x2 and D is 2x3. Resulting matrix will be a 2x3 matrix.
$B \cdot D = \begin{pmatrix} 4 & 2 \\ 5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & -2 & 7 \\ 8 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 \cdot 3 + 2 \cdot 8 & 4 \cdot -2 + 2 \cdot 1 & 4 \cdot 7 + 2 \cdot 2 \\ 5 \cdot 3 + 1 \cdot 8 & 5 \cdot -2 + 1 \cdot 1 & 5 \cdot 7 + 1 \cdot 2 \end{pmatrix} = \begin{pmatrix} 28 & -6 & 32 \\ 23 & -9 & 37 \end{pmatrix}$

1.4 $D \cdot B$

Not Possible, D is 2x3 and B is 2x2.

1.5 $E \cdot F$

Possible, E is 2x1 and F is 1x2. Resulting matrix will be a 2x2 matrix.
$E \cdot F = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 \end{pmatrix} = \begin{pmatrix} 3 \cdot 2 & 3 \cdot -3 \\ 8 \cdot 2 & 8 \cdot -3 \end{pmatrix} = \begin{pmatrix} 6 & -9 \\ 16 & -24 \end{pmatrix}$

1.6 $F \cdot F$

Not Possible, F is 1x2 and not a square matrix.

1.7 $B \cdot G$

Possible, B is 2x2 and G is 2x2. Resulting matrix will also be a 2x2 matrix.
$B \cdot G = \begin{pmatrix} 4 & 2 \\ 5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 4 \cdot 2 + 2 \cdot -3 & 4 \cdot 1 + 2 \cdot 4 \\ 5 \cdot 2 + 1 \cdot -3 & 5 \cdot 1 + 1 \cdot 4 \end{pmatrix} = \begin{pmatrix} 2 & 12 \\ 7 & 9 \end{pmatrix}$

1.8 $G \cdot B$

Possible, G is 2x2 and B is 2x2. Resulting matrix will be a 2x2 matrix. Note that this will not be the same as $B \cdot G$.

$$G \cdot B = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 4 & 2 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 4 + 1 \cdot 5 & 2 \cdot 2 + 1 \cdot 1 \\ -3 \cdot 4 + 4 \cdot 5 & -3 \cdot 2 + 4 \cdot 1 \end{pmatrix} = \begin{pmatrix} 13 & 5 \\ 8 & -2 \end{pmatrix}$$

1.9 $H \cdot D$

Not Possible, H is 3x3 and D is 2x3.

1.10 $C \cdot H$

Possible, C is 3x3 and H is 3x3. Resulting matrix will also be a 3x3 matrix.

$$C \cdot H = \begin{pmatrix} 2 & 5 & 2 \\ 4 & -3 & 3 \\ 2 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 1 \\ 2 & 4 & -4 \\ -3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 + 5 \cdot 2 + 2 \cdot -3 & 2 \cdot -2 + 5 \cdot 4 + 2 \cdot 5 & 2 \cdot 1 + 5 \cdot -4 + 2 \cdot 2 \\ 4 \cdot 1 - 3 \cdot 2 + 3 \cdot -3 & 4 \cdot -2 - 3 \cdot 4 + 3 \cdot 5 & 4 \cdot 1 - 3 \cdot -4 + 3 \cdot 2 \\ 2 \cdot 1 + 1 \cdot 2 + 1 \cdot -3 & 2 \cdot -2 + 1 \cdot 4 + 1 \cdot 5 & 2 \cdot 1 + 1 \cdot -4 + 1 \cdot 2 \end{pmatrix} =$$

$$\begin{pmatrix} 6 & 26 & -14 \\ -11 & -5 & 22 \\ 1 & 5 & 0 \end{pmatrix}$$