Matrices lesson 6 - inverse of a matrix

Magic Monk Tutorials

1 Verify that the matrix B is the inverse of A.

$A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$

$A \cdot B = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 3 \cdot 5 + 2 \cdot -7 & 3 \cdot -2 + 2 \cdot 3 \\ 7 \cdot 5 - 7 \cdot 5 & 7 \cdot -2 + 5 \cdot 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2 Calculate the inverses of the following matrices:

2.1 $\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}^{-1} = \frac{1}{2 \cdot 4 - 7 \cdot 1} \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$

2.2 $\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{1 \cdot 4 - 3 \cdot 1} \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$

2.3 $\begin{pmatrix} 5 & 3 \\ 3 & 3 \end{pmatrix}^{-1} = \frac{1}{5 \cdot 3 - 3 \cdot 3} \begin{pmatrix} 3 & -3 \\ -3 & 5 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 3 & -3 \\ -3 & 5 \end{pmatrix}$

3 Can you calculate the inverse of the following matrix? Why or why not?

$\begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$

The matrix can be inverted if $\frac{1}{ad - bc}$ is defined. In this case, $\frac{1}{ad - bc} = \frac{1}{3 \cdot 4 - 6 \cdot 2} = \frac{1}{12 - 12} = \frac{1}{0}$ which is undefined. Therefore you cannot invert this matrix.

4 Verify that the matrix B is the inverse of A.

$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

$A \cdot B = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 7 + 2 \cdot -2 + 2 \cdot -1 & 1 \cdot -2 + 2 \cdot 1 + 2 \cdot 0 & 1 \cdot -2 + 2 \cdot 0 + 2 \cdot 1 \\ 2 \cdot 7 + 5 \cdot -2 + 4 \cdot -1 & 2 \cdot -2 + 5 \cdot 1 + 4 \cdot 0 & 2 \cdot -2 + 5 \cdot 0 + 4 \cdot 1 \\ 1 \cdot 7 + 2 \cdot -2 + 3 \cdot -1 & 1 \cdot -2 + 2 \cdot 1 + 3 \cdot 0 & 1 \cdot -2 + 2 \cdot 0 + 3 \cdot 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$