Matrices lesson 7 - Solving matrix equations

Magic Monk Tutorials

1 Rearrange for the square matrix X, where all other matricies are also square.

e.g. $AX = B$ solved for X is $X = A^{-1}B$

1.1 $XA = B$

$XA = B$
$XAA^{-1} = BA^{-1}$
$X = BA^{-1}$

1.2 $A(X + B) = C$

$A(X + B) = C$
$A^{-1}A(X + B) = A^{-1}C$
$X + B = A^{-1}C$
$X = A^{-1}C - B$

1.3 $AXB = C$

$AXB = C$
$A^{-1}AXB = A^{-1}C$
$XBB^{-1} = A^{-1}CB^{-1}$
$X = A^{-1}CB^{-1}$

1.4 $AX + BX = C$ (remember the distributitive law)

$AX + BX = C$
$(A + B)X = C$
$X = (A + B)^{-1}\,C$

2 Solve for X in the following, given matrices A, B, and C.

$A = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$

2.1 $AX = B$

$AX = B$ solved for X is $X = A^{-1}B$.
The inverse of A has already been calculated in lesson 6 worksheet question 2.2
$A^{-1} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$
Therefore $X = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} = \begin{pmatrix} 4 \cdot 3 - 1 \cdot 7 & 4 \cdot 2 - 1 \cdot 5 \\ -3 \cdot 3 + 1 \cdot 7 & -3 \cdot 2 + 1 \cdot 5 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$

2.2 $XA = B$

$AX = B$ solved for X is $X = BA^{-1}$.
$A^{-1} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$
Therefore $X = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \cdot \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 3 \cdot 4 + 2 \cdot -3 & 3 \cdot -1 + 2 \cdot 1 \\ 7 \cdot 4 + 5 \cdot -3 & 7 \cdot -1 + 5 \cdot 1 \end{pmatrix} = \begin{pmatrix} 6 & -1 \\ 13 & -2 \end{pmatrix}$

2.3 $XB = A$

$XB = A$ solved for X is $X = AB^{-1}$.
The inverse of B has already been calculated in lesson 6 worksheet question 1.
$B^{-1} = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$
Therefore $X = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 1 \cdot 5 + 1 \cdot -7 & 1 \cdot -2 + 1 \cdot 3 \\ 3 \cdot 5 + 4 \cdot -7 & 3 \cdot -2 + 4 \cdot 3 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -13 & 6 \end{pmatrix}$

2.4 $AXB = C$

$AXB = C$ solved for X is $X = A^{-1}CB^{-1}$.
$A^{-1} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$
$B^{-1} = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$
Therefore $X = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 4 \cdot 2 - 1 \cdot 7 & 4 \cdot 1 - 1 \cdot 4 \\ -3 \cdot 2 + 1 \cdot 7 & -3 \cdot 1 + 1 \cdot 4 \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} =$
$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$