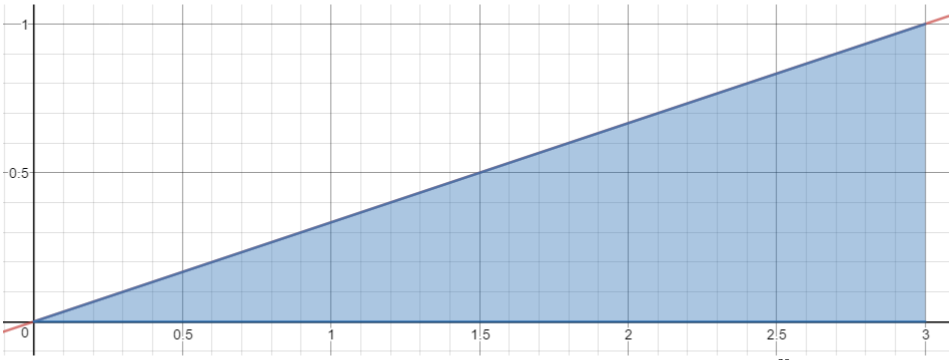
Integral Calculus lesson 4 - Finding the area under a curve

Magic Monk Tutorials

Rule 1: $\int ax^n \, dx = \dfrac{ax^{n+1}}{n+1} + c.$

1 Evaluate the following:

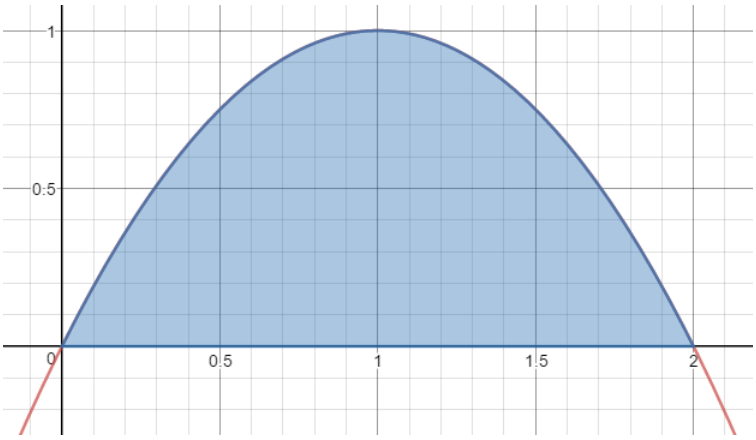
1.1 The shaded area of the curve (i.e. between 0 and 3) $y = \dfrac{x}{3}$



Write this question in integral form. The function is $y = \dfrac{x}{3}$ and we are finding the area between $x = 0$ and 3.

Area $= \displaystyle\int_0^3 \frac{x}{3} \, dx = \left[\frac{x^2}{3*2}\right]_0^3 = \left[\frac{3^2}{6}\right] - \left[\frac{0^2}{6}\right] = \frac{9}{6} = \frac{3}{2}$ units squared.

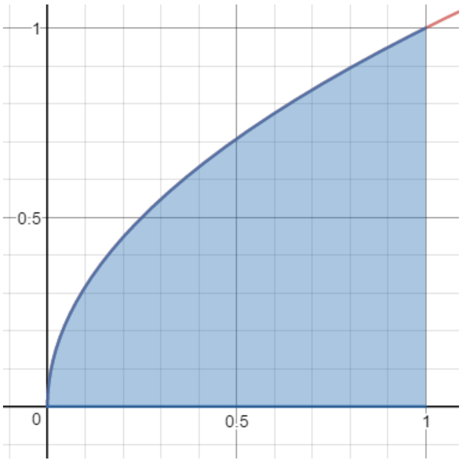
1.2 The shaded area of the curve $y = 2x - x^2$



From the graph, note that the bounds we are integrating over are $x = 0$ and 2 and the function is $y = 2x - x^2$. Write this question in integral form.

Area $= \displaystyle\int_0^2 2x - x^2 \, dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = \left[2^2 - \frac{2^3}{3}\right] - \left[0^2 - \frac{0^3}{3}\right] = 4 - \frac{8}{3} = \frac{4}{3}$ units squared.

1.3 The shaded area of the curve $y = \sqrt{x}$



Note from the diagram the shaded area is between $x = 0$ and 1.

Area $= \int_0^1 \sqrt{x}\ dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 = \left[\frac{2}{3}\cdot 1^{\frac{3}{2}}\right] - \left[\frac{2}{3}\cdot 0^{\frac{3}{2}}\right] = \frac{2}{3}$ units squared.

2 Evaluate the following integrals:

2.1 $\int_0^1 3x\ dx$

$$\int_0^1 3x\ dx = \left[\frac{3x^2}{2}\right]_0^1 = \left[\frac{3\cdot 1^2}{2}\right] - \left[\frac{3\cdot 0^2}{2}\right] = \frac{3}{2}$$

2.2 $\int_0^2 x^2 + x\ dx$

$$\int_0^2 x^2 + x\ dx = \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2 = \left[\frac{2^3}{3} + \frac{2^2}{2}\right] - \left[\frac{0^3}{3} + \frac{0^2}{2}\right] = \frac{8}{3} + \frac{4}{2} = \frac{14}{3}$$

2.3 $\int_{-1}^0 (x+1)^2\ dx$

$$\int_{-1}^0 (x+1)^2\ dx = \left[\frac{(x+1)^3}{3}\right]_{-1}^0 = \left[\frac{(0+1)^3}{3}\right] - \left[\frac{(-1+1)^3}{3}\right] = \frac{1}{3}$$

2.4 $\int_1^2 1\ dx$

$$\int_1^2 1\ dx = [x]_1^2 = [2] - [1] = 1$$