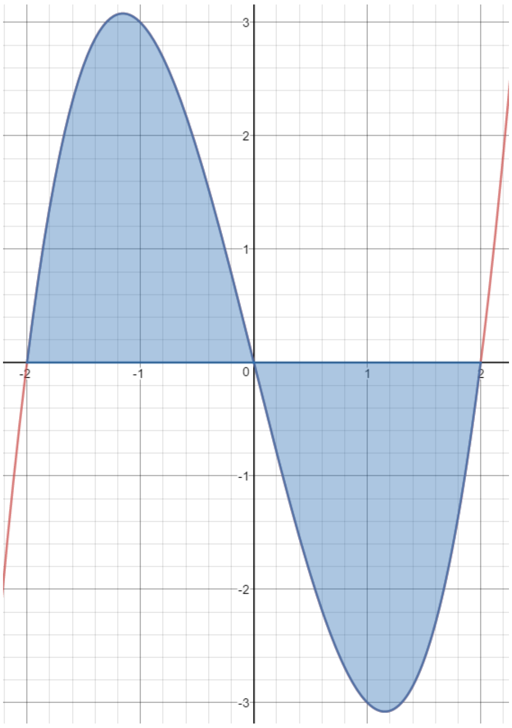
Integral Calculus lesson 6 - Area bounded by the curve pt3

Magic Monk Tutorials

1 Evaluate the following:

1.1 The shaded area of the curve (i.e. between x=-2 and 2) $y = x^3 - 4x$



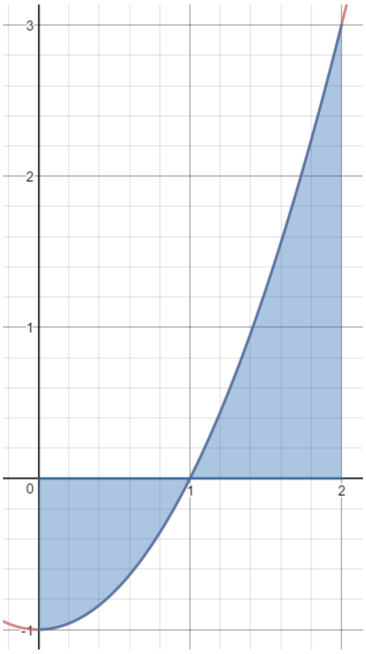
It is clear from the graph that the function is symmetric about the y axis in a sense, and because of this the area of the left shaded region will be the same as the area of the right shaded region. Notice that the function $f(x) = x^3 - 4x$ satisfies $f(x) = -f(-x)$ as can be easily checked.
Therefore, the following is true where the absolute value of both integrals are taken. The graph shows that the first half of the shaded area is from $x = -2$ to 0 and the second half is from $x = 0$ to 2

$$\text{Area} = \int_{-2}^{0} x^3 - 4x\ dx + \int_{0}^{2} x^3 - 4x\ dx = 2\int_{-2}^{0} x^3 - 4x\ dx$$

$$2\int_{-2}^{0} x^3 - 4x\ dx = 2\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} = 2\left[\frac{0^4}{4} - 2\cdot 0^2\right] - 2\left[\frac{(-2)^4}{4} - 2\,(-2)^2\right] = -2\left[\frac{16}{4} - 2\cdot 4\right] = -2\,[-4] = 8$$

Therefore the area is 8 units squared.

1.2 The shaded area of the curve (between x=0 and 2)$y = x^2 - 1$



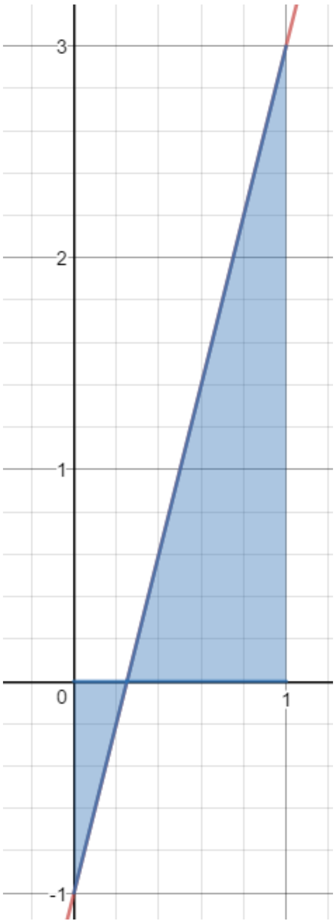
From the graph, it is clear that the function $y = x^2 - 1$ intercepts the x axis at $x = 1$. The area is then the sum of two integrals, where the positvie area is taken of both integrals.

$$\int_0^2 x^2 - 1\ dx = \int_0^1 x^2 - 1\ dx + \int_1^2 x^2 - 1\ dx = \left[\frac{x^3}{3} - x\right]_0^1 + \left[\frac{x^3}{3} - x\right]_1^2$$
$$= \left[\frac{1}{3} - 1 - \frac{0}{3} - 0\right] - \left[\frac{8}{3} - 2 - \frac{1}{3} + 1\right] = \frac{-2}{3} + \frac{4}{3}$$

Take absolute value of the left integral to find the area

Area $= \dfrac{2}{3} + \dfrac{4}{3} = 2$ units squared.

1.3 The shaded area of the curve $y = 4x - 1$



First find the intercepts of the x axis by substituting $y = 0$.
$0 = 4x - 1$
$x = \dfrac{1}{4}$

Split up the integral, noting from the diagram that the shaded area is from $x = 0$ to $x = 1$.

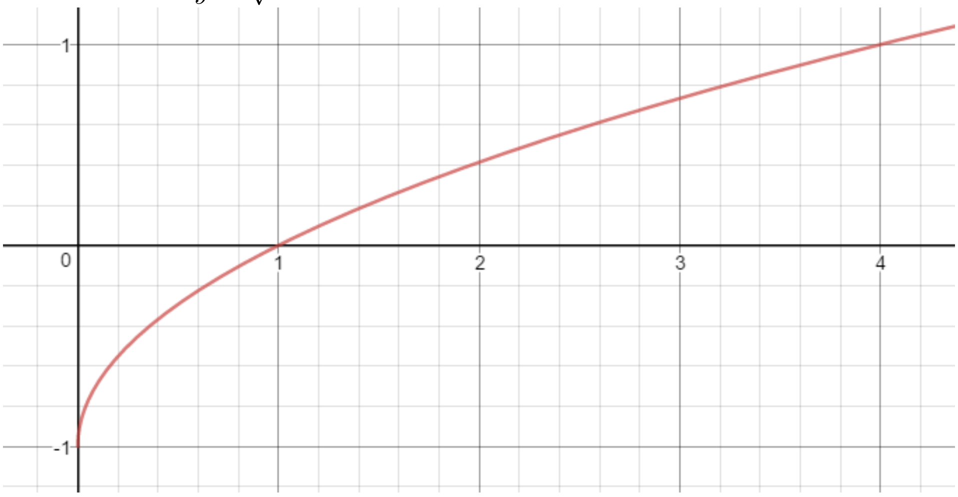
$$\int_0^1 4x{-}1\ dx = \int_0^{\frac{1}{4}} 4x{-}1\ dx + \int_{\frac{1}{4}}^1 4x{-}1\ dx = \left[2x^2 - x\right]_0^{\frac{1}{4}} + \left[2x^2 - x\right]_{\frac{1}{4}}^1 = \left[2\cdot\frac{1}{4}^2 - \frac{1}{4} - 2\cdot 0^2 + 0\right] + \left[2\cdot 1^2 - 1 - 2\cdot\frac{1}{4}^2 + \frac{1}{4}\right] = \left[\frac{1}{8} - \frac{1}{4}\right] + \left[1 - \frac{1}{8} + \frac{1}{4}\right] = \frac{-1}{8} + \frac{9}{8}$$

Take absolute value of the left integral to find the area

Area $= \dfrac{1}{8} + \dfrac{9}{8} = \dfrac{5}{4}$ units squared.

2 Find the area of the curve $y = \sqrt{x} - 1$ between 0 and 4. Remember to plot the curve first, and Keep in mind that this curve crosses the x axis at x=1.

Plot the curve $y = \sqrt{x} - 1$.



The curve intersects the x axis at $x = 1$. Split up the integral as before.

$$\int_0^4 \sqrt{x} - 1\ dx = \int_0^1 \sqrt{x} - 1\ dx + \int_1^4 \sqrt{x} - 1\ dx = \left[\frac{2}{3}x^{\frac{3}{2}} - x\right]_0^1 + \left[\frac{2}{3}x^{\frac{3}{2}} - x\right]_1^4$$
$$= \left[\frac{2}{3} \cdot 1^{\frac{3}{2}} - 1 - \frac{2}{3} \cdot 0^{\frac{3}{2}} + 0\right] + \left[\frac{2}{3} \cdot 4^{\frac{3}{2}} - 4 - \frac{2}{3} \cdot 1^{\frac{3}{2}} + 1\right] = \frac{-1}{3} + \left[\frac{2}{3} \cdot 8 - 3 - \frac{2}{3}\right] = \frac{-1}{3} + \frac{5}{3}$$

Take absolute value of the left integral to find the area

Area $= \frac{1}{3} + \frac{5}{3} = 2$ units squared.