Linear transformations with Matrices lesson 12 - Given the image, find the original point

Magic Monk Tutorials

1 A point (x, y) has been reflected in the x axis with an image of $(2, 3)$. Find the original point (x, y)

From a previous video, we know that the matrix for reflection in the x axis is

$T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

It can be shown that $T = T^{-1}$. Use the general transformation equation, where we have (x', y') and we want to find (x, y).

$\begin{pmatrix} x' \\ y' \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = T^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

2 A point (x, y) has been transformed under $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ with an image of $(3, 2)$. Find the original point (x, y)

As before, find the inverse of T.

$T^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

Use this in our general formula.

$\begin{pmatrix} x' \\ y' \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = T^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3 The line $y' = \frac{1}{2}x' + 2$ is the result of a transformation under the matrix $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Find the original line.

Assume the original line is of the form $y = ax + b$. Apply our transformation T to this line.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + y \\ y \end{pmatrix}$$

Therefore $y' = y$ and $x = x' - y'$. Plug these into our equation $y = ax + b$.

$$y' = a(x' - y') + b$$
$$y' = ax' - ay' + b$$
$$y' + ay' = ax' + b$$
$$y'(1 + a) = ax' + b$$
$$y' = \frac{ax' + b}{1 + a}$$

We may now solve for our a and b, knowing that $2 = \frac{b}{1+a}$ and $\frac{1}{2} = \frac{a}{1+a}$. Starting with our second equation, we have
$1 + a = 2a$ and so $1 = a$. Substituting this into our first equation, we bet $b = 4$. Therefore our original line is $y = x + 4$.