Matrices lesson 12 - Gaussian Elimination part 2

Magic Monk Tutorials

1 Solve the following simultaneous equations with Gaussian Elimination.

1.1

$x + y + z = 3$
$2x + 3y + 2z = 2$
$3x + 2y - z = 1$
Put the above simultaneous equation in matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

Use Augmented matrices.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 3 & 2 & 2 \\ 3 & 2 & -1 & 1 \end{array}\right)$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & -1 & -4 & -8 \end{array}\right)$$

$R3 \to R_3 + R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & -4 & -12 \end{array}\right)$$

From row 3, we know that $-4z = -12$ and therefore $z = 3$. From row 2, we know that $y = -4$. Row 1 gives us the equation $x + y + z = 3$, and substituting in our values of z and y we end up with $x = 4$.

1.2

$2x + 4y + 2z = 4$
$3x + 2y + z = 1$
$6x - 5y + 2z = 3$
Put the above simultaneous equation in matrix form.

$$\begin{pmatrix} 2 & 4 & 2 \\ 3 & 2 & 1 \\ 6 & -5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$$

Use Augmented matrices.

$$\left(\begin{array}{ccc|c} 2 & 4 & 2 & 4 \\ 3 & 2 & 1 & 1 \\ 6 & -5 & 2 & 3 \end{array}\right)$$

Note that he first row can be divided by 2 to simplify the working.$R_1 \to R_1/2$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 2 & 1 & 1 \\ 6 & -5 & 2 & 3 \end{array}\right)$$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 6R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -4 & -2 & -5 \\ 0 & -17 & -4 & -9 \end{array}\right)$$

$R_3 \to R_3 - \frac{17}{4}R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -4 & -2 & -5 \\ 0 & 0 & -4 + 17/2 & -9 + 85/4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -4 & -2 & -5 \\ 0 & 0 & 9/2 & 49/4 \end{array}\right)$$

From row 3, we know that $9/2z = 49/4$ and therefore $z = 49/18$. From row 2, we know that $4y + 2z = 5$ and substituting z, we have $4y + 2 \cdot 49/18 = 5$ therefore $y = -1/9$. From row 1 we have $x + 2y + 1 = 2$, $x - 2/9 + 49/18 = 2$ and then $x = -1/2$.