

Matrices lesson 13 - Solving matrix equations with Gaussian Elimination

Magic Monk Tutorials

1 Solve for X in the following using Gaussian Elimination.

1.1

$$\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$$

Augment the above matrix equation and then rearrange until the left is the identity matrix.

$$\left(\begin{array}{cc|cc} 1 & 1 & 3 & 2 \\ 3 & 4 & 7 & 5 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$\left(\begin{array}{cc|cc} 1 & 1 & 3 & 2 \\ 0 & 1 & -2 & -1 \end{array} \right)$$

$$R_1 \rightarrow R_1 - R_2$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 5 & 3 \\ 0 & 1 & -2 & -1 \end{array} \right)$$

Therefore the matrix X is the right hand matrix.

$$X = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$$

1.2

$$\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} X = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$$

Augment the above matrix equation and then rearrange until the left is the identity matrix.

$$\left(\begin{array}{cc|cc} 3 & 2 & 1 & 1 \\ 7 & 5 & 3 & 4 \end{array} \right)$$

$$R_1 \rightarrow 7R_1$$

$$R_2 \rightarrow 3R_2$$

$$\left(\begin{array}{cc|cc} 21 & 14 & 7 & 7 \\ 21 & 15 & 9 & 12 \end{array} \right)$$

$$R_2 \rightarrow R_2 - R_1$$

$$\left(\begin{array}{cc|cc} 21 & 14 & 7 & 7 \\ 0 & 1 & 2 & 5 \end{array} \right)$$

$$R_1 \rightarrow R_1/7$$

$$\left(\begin{array}{cc|cc} 3 & 2 & 1 & 1 \\ 0 & 1 & 2 & 5 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\left(\begin{array}{cc|cc} 3 & 0 & -3 & -9 \\ 0 & 1 & 2 & 5 \end{array} \right)$$

$$R_1 \rightarrow R_1/3$$

$$\left(\begin{array}{cc|cc} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 5 \end{array} \right)$$

Therefore the matrix X is the right hand matrix.

$$X = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$$

1.3

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 1 & 2 & 3 \end{pmatrix} X = \begin{pmatrix} 3 & 3 & 2 \\ 2 & 3 & 4 \\ 1 & 2 & 5 \end{pmatrix}$$

Augment the above matrix equation and then rearrange until the left is the identity matrix.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 3 & 3 & 2 \\ 2 & 5 & 4 & 2 & 3 & 4 \\ 1 & 2 & 3 & 1 & 2 & 5 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 3 & 3 & 2 \\ 0 & 1 & 0 & -4 & -3 & 0 \\ 0 & 0 & 1 & -2 & -1 & 3 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 11 & 9 & 2 \\ 0 & 1 & 0 & -4 & -3 & 0 \\ 0 & 0 & 1 & -2 & -1 & 3 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 2R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 15 & 11 & -4 \\ 0 & 1 & 0 & -4 & -3 & 0 \\ 0 & 0 & 1 & -2 & -1 & 3 \end{array} \right)$$

Therefore the matrix X is the right hand matrix.

$$X = \begin{pmatrix} 15 & 11 & -4 \\ -4 & -3 & 0 \\ -2 & -1 & 3 \end{pmatrix}$$