

Matrices lesson 14 - Inverse of a 3x3 with Gaussian Elimination

Magic Monk Tutorials

1 Calculate the inverses of the following 3x3 matrices.

1.1

$$\begin{pmatrix} 3 & 3 & -2 \\ 2 & 5 & 4 \\ 1 & 2 & 1 \end{pmatrix}$$

Augment the above matrix with the identity matrix and then rearrange until the left is the identity matrix. The right matrix will then be the inverse.

$$\left(\begin{array}{ccc|ccc} 3 & 3 & -2 & 1 & 0 & 0 \\ 2 & 5 & 4 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$R_1 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 2 & 5 & 4 & 0 & 1 & 0 \\ 3 & 3 & -2 & 1 & 0 & 0 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & -2 \\ 0 & -3 & -5 & 1 & 0 & -3 \end{array} \right)$$

$$R_3 \rightarrow R_3 + 3R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & -2 \\ 0 & 0 & 1 & 1 & 3 & -9 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_3$$

$$R_1 \rightarrow R_1 - R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -1 & -3 & 10 \\ 0 & 1 & 0 & -2 & -5 & 16 \\ 0 & 0 & 1 & 1 & 3 & -9 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 7 & -22 \\ 0 & 1 & 0 & -2 & -5 & 16 \\ 0 & 0 & 1 & 1 & 3 & -9 \end{array} \right)$$

X is now the right hand matrix.

$$X = \begin{pmatrix} 3 & 7 & -22 \\ -2 & -5 & 16 \\ 1 & 3 & -9 \end{pmatrix}$$

1.2

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 3 & 4 & 5 \end{pmatrix}$$

Augment the above matrix with the identity matrix and then rearrange until the left is the identity matrix. The right matrix will then be the inverse.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 3 & 3 & 0 & 1 & 0 \\ 3 & 4 & 5 & 0 & 0 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & -2 & -1 & -3 & 0 & 1 \end{array} \right)$$

$$R_2 \rightarrow -1 \cdot R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & -2 & -1 & -3 & 0 & 1 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$R_3 \rightarrow R_3 + 2R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 - R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array} \right)$$

X is now the right hand matrix.

$$X = \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & -1 \\ 1 & -2 & 1 \end{pmatrix}$$

1.3

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 2 \\ 2 & 4 & -7 \end{pmatrix}$$

Augment the above matrix with the identity matrix and then rearrange until the left is the identity matrix. The right matrix will then be the inverse.

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 2 & 4 & -7 & 0 & 0 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 2 & -11 & -2 & 0 & 1 \end{array} \right)$$

$$R_1 \rightarrow R_1 - R_2$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 3 & -1 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -7 & 2 & -2 & 1 \end{array} \right)$$

$$R_3 \rightarrow R_3 / -7$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 3 & -1 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2/7 & 2/7 & -1/7 \end{array} \right)$$

$$R_2 \rightarrow R_2 + 2R_3$$

$$R_1 \rightarrow R_1 - 4R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 + 8/7 & -1 - 8/7 & -4/7 \\ 0 & 1 & 0 & -2 - 4/7 & 1 + 4/7 & -2/7 \\ 0 & 0 & 1 & -2/7 & 2/7 & -1/7 \end{array} \right)$$

X is now the right hand matrix. Simplify this matrix further.

$$\begin{aligned} X &= \begin{pmatrix} 3 + 8/7 & -1 - 8/7 & -4/7 \\ -2 - 4/7 & 1 + 4/7 & -2/7 \\ -2/7 & 2/7 & -1/7 \end{pmatrix} \\ &= \begin{pmatrix} 29/7 & -15/7 & -4/7 \\ -18/7 & 11/7 & -2/7 \\ -2/7 & 2/7 & -1/7 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 29 & -15 & -4 \\ -18 & 11 & -2 \\ -2 & 2 & -1 \end{pmatrix} \end{aligned}$$