Matrices lesson 8 - Solving sim. equations with matrices

Magic Monk Tutorials

1 Solve the following simultaneous equations with matrices.

1.1

$x + y = 2$
$x - y = 1$

Put the above simultaneous equation in matrix form.

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Calculate the inverse of $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{-1-1} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix}$$

Therefore, by rearranging our initial matrix equation,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + 1/2 \\ 1 - 1/2 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 1/2 \end{pmatrix}$$

Therefore $x = 3/2$ and $y = 1/2$ solves the simultaneous equations.

1.2

$3x + 2y = 4$
$x + 2y = 1$

Put the above simultaneous equation in matrix form.

$$\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Calculate the inverse of $\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$.

$$\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \frac{1}{6-2} \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 \\ -1/4 & 3/4 \end{pmatrix}$$

Therefore, by rearranging our initial matrix equation,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 \\ -1/4 & 3/4 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 - 1/2 \\ -1 + 3/4 \end{pmatrix} = \begin{pmatrix} 3/2 \\ -1/4 \end{pmatrix}$$

Therefore $x = 3/2$ and $y = -1/4$ solves the simultaneous equations.

1.3

$2x + y = 3$
$7x + 4y = 2$

Put the above simultaneous equation in matrix form.

$$\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Calculate the inverse of $\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$.

$$\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}^{-1} = \frac{1}{8-7} \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

Therefore, by rearranging our initial matrix equation,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 12-2 \\ -21+4 \end{pmatrix} = \begin{pmatrix} 10 \\ -17 \end{pmatrix}$$

Therefore $x = 10$ and $y = -17$ solves the simultaneous equations.

2 Solve the following simultaneous equations, given the following information.

$x + 2y + 2z = 2$
$2x + 5y + 4z = 3$
$x + 2y + 3z = 4$
and

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 1 & 2 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Begin by putting the simultaneous equations in matrix form. We will need a 3x3 matrix for the coefficients, a 3x1 matrix for the variables, and 3x1 for the constants.

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

We have been given the inverse of the first matrix. We can now rearrange for x,y,z.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 & -2 & -2 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 14-6-8 \\ -4+3 \\ -2+4 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$$

Therefore $x = 0$, $y = -1$ and $z = 2$ solves the given simultaneous equations.